

# When Candidates and Vote Distribution Matter: A New Indicator of Electoral Competitiveness

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The electoral competitiveness among candidates vying for single elected positions (e.g. president, members of parliament single member districts, or candidates for the party leadership) lacks an appropriate measurement. This study reevaluates previous measurements and proposes a new indicator that accounts for the interaction between the number of candidates and the distribution of votes. The resulting indicator overcomes the oversensitivity problem associated with earlier specification and provides better competitiveness estimate for various electoral settings. Its applicability is universal and allows for cross-cases and longitudinal comparisons for a wide variety of single-winner elections.

**Key words:** electoral competition; single winner; candidates; vote distribution.

## 1. INTRODUCTION

Political competition lies at the heart of contemporary representative democracies. Its effects on a large number of electoral and political processes lead to a series of attempts to develop valid and reliable measures. Whereas the electoral competitiveness between political parties has been approached differently in various studies (Key 1949; Mayhew 1974; Ranney 1976; Bibby 1990; Przeworski 1991; Holbrook and van Dunk 1993; Vanhanen 1997; Hall 2001; Besley and Preston 2002; Bibby and Holbrook 2001), considerable less attention has been devoted to the competition between individual candidates for a single post. A few

examples of competitions falling in this category are the presidential and local elections (i.e. the mayor), elections in single member districts, or intra-party leadership selection.

When referring to competitiveness, Mayhew (1974) employed the margin between the two largest parties. Adapted to individual contexts, the competitiveness is reflected by the margin between the two main candidates. Similarly, the measurement proposed by Vanhanen (1997) for the electoral competition between political parties can be transposed in the setting of elections among individuals: the competitiveness is given by the share of the vote won by the most popular candidate. The major shortcoming of both indicators is that they ignore the number of candidates and the distribution of votes among candidates other than the first two in the case of Mayhew and the most popular candidate for Vanhanen.

Two supplementary indicators were reviewed by Kenig (2009): the incumbents' success rate and the likelihood of contests (direct coronations). Their weaknesses have been carefully revealed by the author: the incumbents' success rate refers only to contests in which incumbents participate and where challengers are formally organized, whereas the coronations differentiates between single-candidate and multi-candidate instances (Kenig 2009, 244-245). To overcome these shortcomings, Kenig constructs an index – effective number of candidates (ENC) – based on the commonly used effective number of parties (ENP) index (Laakso and Taagepera 1979). And he incorporates “the absolute number of candidates” (denoted by  $N$ ) into the *ENC* formula as the denominator to reflect “the extent to which the number of candidates had shrunk” (electoral competitiveness) (Kenig 2009, 246).

Although this is so far the most sophisticated solution to account for competition for a single post developed, as we will demonstrate shortly, simulation analysis suggests that this index suffers from sensitivity (to the number of candidates) and identification problem resulting from  $N$ 's larger marginal effects and the upper bound limit imposed by Kenig's model specification. This article proposes a new competitiveness index that alleviates these problems and improves the estimation precision through more efficient use of vote distribution in formation. Our competitiveness indicator is thus robust to and facilitates comparison of various kinds of electoral setting that involve different number of candidates and diverse vote distribution patterns in single-winner elections.

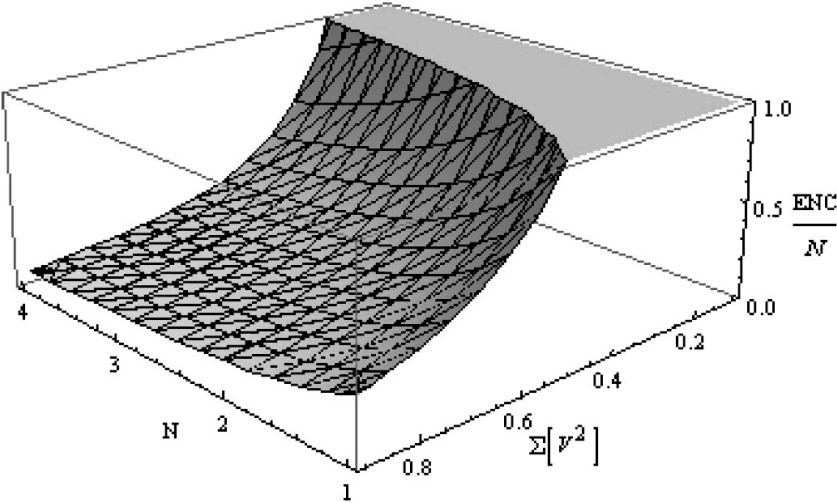
## 2. PROBLEMS WITH THE *ENC/N* INDEX

*ENC/N* is a decreasing function of the interaction term  $N * \sum V_i^2$ . Kenig added an  $N$  term to normalize his *ENC* index on a 0-1 scale with an aim to “reflect the distribution of votes and [will] neutralize the effect of the absolute number of candidates”; however, only scant attention is paid to the behavior of the marginal

effects of this interaction term when its two components,  $N$  and  $\Sigma V_i^2$ , are evaluated at the opposite extremes.

Figure 1 shows how the curvature of the  $ENC/N$  index surface changes when  $\Sigma V_i^2$  and  $N$  are evaluated across their full range of values.<sup>1</sup> At higher levels of  $\Sigma V_i^2$  (higher concentration of votes in a few candidates), smaller  $N$  makes the  $ENC/N$  index more competitive, as exhibited in the quick “jump” in curvature in the direction of  $N$  when the absolute value of  $N$  falls under 2—a phenomenon which is empirically unsound. In addition, note that as  $\Sigma V_i^2$  approaches 0, the  $ENC/N$  index shoots up to its maximum value (1) regardless of the parameter value on the  $N$ -axis, as shown in the light-colored trapezoid-shape area in the upper right corner of this plot, causing identification problem in this area when one tries to compare the relative competitiveness among contests with different size of  $N$ . Apparently, Kenig’s specification fails to achieve its intended purposes.

Figure 1  $ENC/N$  as a Function of  $\Sigma V_i^2$  and  $N$



We argue that these biases are rooted in the multiplicative nature of the  $ENC/N$  index’s denominator term. First, recall that Kenig’s  $ENC/N$  index is just the inverse of the multiplicative term  $N * \Sigma V_i^2 \forall i = 1, \dots, N$ . Theoretically speaking, as  $N$  increases, at the margin, it tends to pull votes away from major contenders because more candidates always present more alternatives for selectorates, which makes the distribution of votes *less* concentrated. According to Kenig (2009, 236), a more equal distribution of votes would make the contests *more* competitive.

However,  $ENC/N$  is decreasing in both  $N$  and  $\Sigma V_i^2$  through the composite denominator term  $N * \Sigma V_i^2$ , and in Kenig’s specification  $N$  is any integers  $\geq 2$  while  $\Sigma V_i^2$  is bounded between 0 and 1,<sup>2</sup> it is easy to see that the marginal effect of  $N$  drives down the  $ENC/N$  score faster than  $\Sigma V_i^2$ . When we compare two elections with similar vote distribution patterns, the election with more fringe candidates will receive much lower  $ENC/N$  estimate owing to  $N$ ’s much larger diminishing marginal effect than that of  $\Sigma V_i^2$ .

Even in instances with the same number of candidates the  $ENC/N$  index is slightly problematic as it is influenced by the share of votes received by the winner. If this is small, then the index is large (see  $H_1$  and  $H_4$ ). Apart from this, his index is quite sensitive to the vote share received by small competitors and to the number of competitors (at the expense of difference between the first two candidates). A practical illustration of these shortcomings is reflected in the four hypothetical situations listed in Table 1. Elections  $H_1$  and  $H_2$  illustrate this issue: the vote share difference between the first three candidates is similar, the share of the winner is smaller in  $H_1$ ; At the same time, the share of the least popular candidate is higher. Consequently,  $H_1$ ’s competitiveness index goes up. Elections  $H_3$  and  $H_4$  show how the index overestimates the diminishing marginal effect exerted by small competitors. The presence of fringe candidates significantly alters the competitiveness index.

**Table 1** Share of Votes, Number of Candidates, and Competitiveness Index

Vote Share	$H_1$	$H_2$	$H_3$	$H_4$
$V_1$	32	33	32	30
$V_2$	28	29	28	28
$V_3$	21	22	21	21
$V_4$	19	16	18	19
$V_5$			1	1
$V_6$				1
ENC/N	0.957	0.936	0.777	0.670

It is worth elaborating the identification problem outlined previously. This issue is related to Kenig’s attempt to transform the unbounded  $ENC$  indicator into a bounded index. This is commendable because such treatment allows the  $ENC/N$  index of the same  $N$ -class to be well-behaved within the 0-1 scale, but it also constrains our ability to extend our competitiveness analysis of elections whose  $N(s)$  range across the full range of possible values. Some simple algebraic

expressions might help clarify this issue. Suppose for a given election, votes are equally distributed among all  $N$  candidates, this election's  $ENC/N$  score is calculated as

$$\frac{1}{N * \sum V_i^2} = \frac{1}{N * \left( N * \left( \frac{1}{N} \right)^2 \right)} = 1$$

implying the maximum level of competitiveness. Note the maximum value is invariant to the size of  $N$ . Although Kenig acknowledged his  $ENC/N$  index “works to its best when the (absolute) number of candidates is equal” (Ibid, 245), one is hard-pressed to accept the notion that, when votes are equally distributed, candidates in a three-way election face the same level of competitiveness as their counterparts in a canonical two-way contest as suggested by their identical maximum value (1).<sup>3</sup> This built-in identification problem presents an estimation bias that needed to be reckoned with.

In sum, when vote distribution statistics are similar across elections, the  $ENC/N$  index tends to underestimate the competitiveness of elections having higher number of candidates due to  $N$ 's larger diminishing marginal effect and when vote shares are perfectly equal among candidates, the index cannot distinguish the relative competitiveness among elections with different number of candidates. In the next section, we propose a remedy that alleviates these biases but retains  $N$ 's desirable property.

### 3. A VARIANCE COMPONENT APPROACH

Drawing on the findings of these studies, we seek to correct the sensitivity issues that have plagued the previous indicators. Improving upon Kenig's  $ENC/N$  index, we address the confounding effect of the interaction between the number of candidates and the distribution of votes on competitiveness estimate through variance component approach. We then formulate a new composite indicator, the Electoral Competitiveness Indicator ( $ECI$ ), with an aim toward providing a more flexible estimator that has the ability to assess competitiveness in a wide range of electoral contexts.

Our task is to find a feasible approach to scale down  $N$ 's marginal effect without altering the diminishing effect it exerts on the  $ENC/N$  index through its interaction with  $\sum V_i^2$ . We want to achieve this without doing too much violence to Kenig's original  $ENC/N$  functional form since it attends appropriately to the interaction between  $N$  and  $\sum V_i^2$ . Our  $ECI$  is composed of two elements, an adjusted  $ENC$  indicator and a pooling factor, each with its specific estimation purposes. The adjusted  $ENC$  indicator, shown in equation (1), dilutes the sensitivity issue by

using the square root of  $N$  in the specification; this specification also overcomes the identification problem caused by Kenig's upper bound normalization as this allows the indicator to vary with  $N$ . A simple solution is simply to take the square root of  $N$  and then plug it into the original formula:

$$Adj \frac{ENC}{N} \equiv \frac{1}{\sqrt{N} * \sum V_i^2}, \forall i = 1, \dots, N \quad (1)$$

Because  $\sqrt{N}$  shrinks the absolute value of  $N$  our adjusted  $ENC/N$  index is thus less sensitive to the marginal change in  $N$ . Also note that our  $Adj-ENC/N$  index will give a maximum value of  $\sqrt{N}$  when votes are equally distributed and this value is monotonically increasing in  $N$ . We regard our unbounded  $Adj \frac{ENC}{N}$  to more faithfully capture the increasing level of competitiveness as a result of more equally-competitive candidates entering the race, as compared to the maximum value estimated by the  $ENC/N$  index which is unconditional on  $N$ .

Our attempt to propose electoral competitiveness indicator (ECI) does not stop here. Although the  $Adj \frac{ENC}{N}$  marks an improvement over the original  $ENC/N$  index in terms of underestimation bias (for elections with large  $N$ ) and identification problem, a competitiveness indicator is only useful when it can be used to evaluate the relative competitiveness of elections that have diverse vote distribution and number of candidates  $\{\sum V_i^2, N\}$  profiles. If the original  $ENC/N$  index works (partially) fine only when the number of candidates is equal, we would like to extrapolate our analysis to instances that beyond this constraint. To this aim, an adjustment factor would be needed to offset the inherent downward bias imposed by the  $N$  term in our  $Adj \frac{ENC}{N}$  indicator.

Empirically, as  $N$  increases, it makes the distribution of votes less concentrated in major candidates which then causes the election to become more competitive; on the contrary, holding  $N$  constant, higher  $\sum V_i^2$  indicates that the distribution of votes is concentrated in a handful of candidates (*i.e.*, large  $\sum V_i^2$ ), which makes the election less competitive. Clearly, two competing effects are at work in influencing electoral competitiveness and their effects are translated through the vote distribution mechanism; however, this mechanism is poorly modeled by the specification of equation (1) because  $ENC/N$  is strictly decreasing in both  $\sum V_i^2$  and  $N$ . The question now comes down to how can we make more efficient use of the information supplied by  $\{\sum V_i^2, N\}$  to improve our competitiveness estimates across cases having diverse  $N$ .

Ideally, we need an adjustment factor that can adjudicate the competing effects of  $\sum V_i^2$  and  $N$  on vote distribution and allow us to use this information to determine the proportion of the variance in electoral competitiveness that should be estimated by the estimator derived in (1). A useful first step toward the construction of such

factor is to conceptualize the relationship between  $\Sigma V_i^2$  and  $N$  as that between variance and sample size  $n$  in a random effects model. Unlike classical regression analysis where the group-level predictors ( $\alpha_i$ ) and regression mean ( $\mu$ ) are collinear,<sup>4</sup> in a random effects model group-level predictors are shrunk toward their own estimated value ( $\hat{\alpha}_i$ ) for groups with more observation ( $n$ ) and when within-group standard deviation ( $\sigma_j$ ) is small, but there is more pooling toward regression mean ( $\mu$ ) when the between-group standard deviation ( $\sigma_\alpha$ ) is small:

$$\alpha_j \approx \left( \frac{\frac{n_j}{\sigma_j^2}}{\frac{n_j}{\sigma_j^2} + \frac{1}{\sigma_\alpha^2}} \right) \hat{\alpha}_j = \left[ 1 - \left( \frac{\frac{n_j}{\sigma_j^2}}{\frac{n_j}{\sigma_j^2} + \frac{1}{\sigma_\alpha^2}} \right) \right] \mu \tag{2}$$

This same logic also applies electoral competitiveness analysis. An increase in smoothes out marginally the *within-election* (or *between-candidate*) vote share variance and shrinks the estimate toward this particular election’s  $Adj \frac{ENC}{N}$  value given by equation (1) ; on the contrary, higher  $\Sigma V_i^2$  implies greater *within-election* vote share variance, which pools the estimate toward the mean estimate ( $\sqrt{N}$ ). By expression (2), we can similarly define a pooling factor for election  $j$  with  $N$  candidates:

$$\varphi \equiv \frac{\frac{N_j}{\Sigma V_{ij}^2}}{\frac{N_j}{\Sigma V_{ij}^2} + \frac{N_j}{N_j \cdot \left(\frac{1}{N_j}\right)^2}} \tag{3}$$

We then multiply expression (3) by equation (1) to denote the proportion of electoral competitiveness to be estimated by election  $j$ ’s unique  $Adj \frac{ENC}{N}$  value, and let  $(1 - \varphi)$  proportion of this competitiveness estimate to be pooled toward the mean estimate ( $\sqrt{N}$ ). Adding these two parts together, we get a weighted competitiveness estimator, *Electoral Competitiveness Indicator (ECI)*:

$$ECI \equiv \varphi \left( \frac{1}{\sqrt{N_j + \Sigma V_{ij}^2}} \right) + (1 - \varphi) \cdot \sqrt{N_j} \tag{4}$$

Clearly, this weighted estimator tends to attribute the original estimator (equation (1)) greater weight whenever  $N_j$  is large but pools toward the election mean when  $\Sigma V_{ij}^2$  increases and these partial pooling effects are translated through the adjustment factor  $\varphi$  because  $\frac{\partial \varphi}{\partial N} > 0$  and  $\frac{\partial \varphi}{\partial (\Sigma V_i^2)} < 0$ .

How does the *ECI* work? Table 2 replicates the same four hypothetical examples ( $H_1$  to  $H_4$ ) from Table 1 to illustrate the operation of our competitiveness indicator,

*ECI*. We complements our analysis with two other hypothetical cases ( $H_5$  to  $H_6$ ) to examine if the weighted *ECI* estimate is robust in instances when votes are distributed perfectly equal among candidates. The comparison of election  $H_1$  to  $H_2$  provides a modal case to elucidate the mechanism of our adjustment factor,  $\varphi$ . The effect of smaller vote share difference between the pair of two weaker candidates,  $V_3$  and  $V_4$ , in  $H_1$  (0.2 versus  $H_2$ 's 0.6) is picked up by  $H_1$ 's smaller  $\Sigma V_i^2$  and translated to its higher  $\varphi$  value, thereby shrinking the estimate toward  $H_1$ 's unique  $Adj \frac{ENC}{N}$ . Because  $H_1$  has higher  $Adj \frac{ENC}{N}$  estimate (1.915709), it therefore receives a higher *ECI* score than  $H_2$ . This exercise shows how the *ECI* uses the information supplied by  $\Sigma V_i^2$  to adjudicate the relative competitiveness between cases.

**Table 2** Share of Votes and Competitiveness of Elections in Six Hypothetical Contexts

Vote Share	$H_1$	$H_2$	$H_3$	$H_4$	$H_5$	$H_6$
$V_1$	0.32	0.33	0.32	0.30	0.2	0.166
$V_2$	0.28	0.29	0.28	0.28	0.2	0.166
$V_3$	0.21	0.22	0.21	0.21	0.2	0.166
$V_4$	0.19	0.16	0.18	0.19	0.2	0.166
$V_5$			0.01	0.01	0.2	0.166
$V_6$				0.01		0.166
$\Sigma V_i^2$	0.261	0.267	0.2574	0.2488	0.2	0.1653
$N$	4	4	5	6	5	6
$Adj \frac{ENC}{N}$	1.915709	1.872659	1.737427	1.640869	2.236068	2.469742
$\varphi$	0.489237	0.483559	0.437254	0.401156	0.500000	0.520400
<i>ECI</i>	1.958762	1.938423	2.018035	2.125107	2.236068	2.557929

We now turn to instances when the elections being compared have different number of candidates. The comparison of  $H_1$  to  $H_3$  highlights the ability of the *ECI* to detect the marginal increase in competitiveness made by the presence of one fringe candidate ( $V_5$ ) in  $H_3$ , the *ECI* gives a higher competitiveness estimate for  $H_3$  through more pooling toward its election mean ( $\sqrt{5}$ , which is higher than  $H_1$ 's ( $\sqrt{4}$ )) despite its having lower  $Adj \frac{ENC}{N}$  score than that of  $H_1$ . Also, when two more fringe candidates were present and absorb a small share of the vote away from the strongest candidate ( $V_1$ ), thus narrowing the vote margin between the two leading candidates ( $V_1$  and  $V_2$ ), a situation typified by scenario  $H_4$ , our *ECI* correctly delivers a higher competitiveness score for  $H_4$  against the baseline scenario  $H_1$  as compared



to the estimates obtained by using the *ENC/N* index in Table 1 which incorrectly gives election  $H_1$  higher competitiveness estimate (0.957854) than  $H_4$  (0.669882). Finally, we look at election  $H_5$  to  $H_6$ , our analysis shows that even under perfectly equal vote share scenario, our ECI still outperforms Kenig's *ENC/N*. *ECI* assigns higher score to elections with larger number of candidates ( $N$ ) as opposed to the *ENC/N* index which is invariant to the marginal change in  $N$  when the index is evaluated at its maximum value. This brief analysis persuasively demonstrates the flexibility and improved precision of our proposed variance component-based estimator in comparing competitiveness of electoral contests with unequal  $N$ .

#### 4. DISCUSSION

This paper proposes a new measurement of electoral competitiveness between candidates running for single posts. In doing so, it re-evaluates extant electoral competitiveness measures through simulations and, by tackling the sensitivity and identification issues that have plagued previous studies, we develop a new estimator, *ECI*. This is robust to the confounding influence of the interaction between the number of candidates and their vote shares. In that respect, our indicator improves the estimation precision of previously developed measures (Kenig 2009) through more efficient use of vote distribution information; it is flexible enough to provide precise comparative competitiveness estimates across elections with varying number and strength of candidates. *ECI* overcomes the methodological problems of the previous measures and proposes a generally valid indicator. This accuracy appears to come at the cost of simplicity as the calculations are slightly more complex than the existing formulas. To compensate for the somewhat technical explanations within the text, Appendix 1 is meant to increase the accessibility of more users and explains how the indicator can be computed.

While the methodological implications of the *ECI* have been clearly outlined in the body of the paper, it provides at least three major empirical benefits. First, the *ECI* is a universal measure for competitiveness in various electoral competitions for a single elected position. It is not sensitive to time or place and allows comparability of a broad range of electoral contests. At the same time, it provides a standardized measure that allows comparability of results across units of analysis over time. On these grounds, the second empirical implication is that *ECI* can be used in a variety of studies ranging from the electoral competitions (e.g. of candidates in single-member districts, for presidential elections) to leadership positions in organizations (i.e. political parties, administration, civil society etc.). Third, it enhances the processes of replication and reliability control. Researchers can use the *ECI* to take a retrospective look at various elections and evaluating their level of competitiveness.

## APPENDIX: HOW TO CALCULATE THE ECI

### 1. Operationalization

To impute the ECI, we need the results of the electoral contests, the number of effective candidates ( $N$ ), and we have to normalize each candidate's vote share ( $V_i$ ) between 0 and 1. To express this concept formally, assuming there are  $N$  candidates

$$i = 1, 2, 3, \dots, n, \text{ and } N > 0,$$

and each candidate receives  $V_i$ 's of total vote in a particular election where:

$$V_i \in [0, 1]^5 \text{ and } \sum_1^n V_i = 1.$$

### 2. Computing the ECI

#### 2.1 The adjusted ENC/N index

This “adjusted” index measures the competitiveness of an election. As noted in the paper, we want this measure to be able to capture the *increasing* competitiveness resulting from more candidates entering the race without this positive effect being overtly diluted by the  $N$  term in the denominator. To alleviate  $N$ 's larger marginal effect (relative to  $\sum_1^n V_i^2$ ), we use the square root of  $N$  in the specification and operationalize our *adjusted ENC/N index* as

$$\frac{ENC}{N} \equiv \frac{1}{\sqrt{N} * \sum_1^n V_i^2}, \forall i = 1, \dots, N. \quad (1)$$

Note that the *adjusted ENC/N index* is less sensitive to the marginal change in  $N$  because the square root scales down the marginal effect of  $N$ . This operationalization procedure has desirable property in the sense that when votes are equally distributed among candidates, as we illustrated in the comparison of scenario  $H_5$  and  $H_6$  in the paper,  $\sqrt{N}$  specification allows the interactive denominator term,  $\sqrt{N} * \sum_1^n V_i^2$ , to vary according to changes in  $N$  and vote shares distribution among candidates. This overcomes the identification problem that plagued Kenig's *ENC/N index*.

#### 2.2 The pooling factor

Another methodological contribution of this study is that we extend the assessment of electoral competitiveness beyond cases with the same number of candidates. We want to re-emphasize here that there are two competing effects at work in influencing electoral competitiveness. An increase in  $N$  flattens vote distribution,

causing elections to be *more* competitive while higher  $\Sigma V_i^2$  makes vote distribution more concentrated, indicating less electoral competitiveness. As we argued in the paper, this mechanism is poorly modeled by the original ENC/N index and the specification of expression (1) since  $Adj \frac{ENC}{N}$  is decreasing in both  $\Sigma V_i^2$  and  $\sqrt{N}$ . To extrapolate our analysis beyond this constraint would require an adjustment factor to offset the downward bias imposed by the  $N$  term without discarding useful “competitiveness” information (i.e., relative vote share among candidates) contained in the interaction term,  $\sqrt{N} * \Sigma_1^n V_i^2$ .

We first need to construct a pooling factor to partial out the vote share variances associated with an election’s (say, election  $j$ ) unique vote share pattern  $\Sigma V_{ij}^2$  from the expected vote share variance of an election with  $N$  effective candidates which is simply this the mean value,  $\frac{1}{N_j}$  (i.e., when each candidate receives equal vote share,  $V_1 = V_2 = \dots = V_n = \frac{1}{N_j}$ , and  $\frac{1}{N_j}$  is unconditional on the value and the distribution of  $V_i$ ). As we have elucidated in our paper, we use  $N_j$  and  $\Sigma V_{ij}^2$  to approximate the effective number of candidates and within-election vote share variance in election  $j$ , respectively. We then operationalize the pooling factor  $\varphi$  as:

$$\varphi \equiv \frac{\frac{N_j}{\Sigma V_{ij}^2}}{\frac{N_j}{\Sigma V_{ij}^2} + \frac{N_j}{N_j * (\frac{1}{N_j})^2}} \quad (3)$$

Where  $\varphi$  determines the amount of vote share variance that are deviated from the expected mean estimate and which should be estimated by election-specific competitiveness estimate,  $Adj \frac{ENC}{N}$ . By (1) and (3), we now specify our *ECI* as a weighted estimator:

$$ECI \equiv \varphi \left( \frac{1}{\sqrt{N_j * \Sigma V_{ij}^2}} \right) + (1 - \varphi) \sqrt{N_j} \quad (4)$$

where  $\sqrt{N}$  is the expected vote share variance which is calculated from  $\frac{1}{\sqrt{N * \Sigma V_i^2}} = \frac{1}{\sqrt{N * (N * (\frac{1}{N})^2)}} = \sqrt{N}$ , by (1) and given condition that  $V_1 = V_2 = \dots = V_n = \frac{1}{N}$ . Note that because  $\frac{\partial \varphi}{\partial N} = \frac{1}{\Sigma V^2} > 0$  and  $\frac{\partial \varphi}{\partial (\Sigma V^2)} = - \frac{1}{(\Sigma V^2)^2} \left( \frac{2}{\Sigma V^2} + N \right) < 0$ ,

$\varphi$  is increasing in  $N_j$  but decreasing in  $\Sigma_1^n V_{ij}^2$ , a larger effective candidate size,  $N$ , pools the *ECI* estimate toward the adjusted *ENC/N* index, while a larger within-

election vote share variance,  $\Sigma V_{ij}^2$ , makes  $\left(\frac{1}{\sqrt{N_j + \Sigma V_{ij}^2}}\right)$  a less precise competitive estimator and therefore pools the *ECI* toward the  $\sqrt{N}$  since the computation of the latter does not depend on the information of  $\Sigma V_{ij}^2$ .

### 3. Application

Now consider the six hypothetical election scenarios from Table 2:

Vote share	$H_1$	$H_2$	$H_3$	$H_4$	$H_5$	$H_6$
$V_1$	0.32	0.33	0.32	0.3	0.2	0.16
$V_2$	0.28	0.29	0.28	0.28	0.2	0.16
$V_3$	0.21	0.22	0.21	0.21	0.2	0.16
$V_4$	0.19	0.16	0.18	0.19	0.2	0.16
$V_5$			0.01	0.01	0.2	0.16
$V_6$				0.01		0.16

We first square the  $V_i$  value in each column and sum them together to obtain the vote share variance for each hypothetical election scenario,  $\Sigma V_{ij}^2$  for  $j = 1$  to 6. We get:

	$H_1$	$H_2$	$H_3$	$H_4$	$H_5$	$H_6$
$\Sigma V_{ij}^2$	0.261	0.267	0.2574	0.2488	0.2	0.1536

We then substitute each election's  $N_j$  and  $\Sigma V_{ij}^2$  into expression (1) to calculate the  $Adj \frac{ENC}{N}$  for each election scenario, which are given below:

	$H_1$	$H_2$	$H_3$	$H_4$	$H_5$	$H_6$
$AdjENC/N$	1.915709	1.872659	1.737427	1.640869	2.236068	2.657867

We now compute the relevant statistics required to estimate  $\phi$ . First we use the information given in each election scenario'  $\{N_j, \Sigma V_{ij}^2\}$  to obtain their  $\frac{N_j}{\Sigma V_{ij}^2}$  and  $\frac{N_j}{N_j * (\frac{1}{N_j})^2}$  (which equals  $N^2$ ). We then use these statistics to calculate  $\phi$ , which is simply the ratio of  $\frac{N_j}{\Sigma V_{ij}^2}$  to the sum of  $\frac{N_j}{\Sigma V_{ij}^2}$  and  $\frac{N_j}{N_j * (\frac{1}{N_j})^2}$  by expression (3).

	$H_1$	$H_2$	$H_3$	$H_4$	$H_5$	$H_6$
$\frac{N_j}{\sum V_{ij}^2}$	15.32567	14.98127	19.42502	24.11576	25	39.0625
$\frac{N_j}{N_j * (\frac{1}{N_j})^2}$	16	16	25	36	25	36
$\phi$	0.489237	0.483559	0.437254	0.401155	0.5	0.5204

Finally, we substitute the values of  $Adj \frac{ENC}{N}$ ,  $\phi$  and the expected (unconditional) vote share variance,  $\sqrt{N}$  into expression (3) to get each election's *ECI* score:

	$H_1$	$H_2$	$H_3$	$H_4$	$H_5$	$H_6$
<i>ECI</i>	1.958762	1.938423	2.018035	2.125107	2.236068	2.557929

**NOTES**

- 1 We set  $N \in (1, 4)$  for illustration purpose.
- 2 The fact that  $\sum v_i^2$  is a squared term means that the effect of vote share on electoral competitiveness has been scaled down.
- 3 We should also point out that when votes are distributed perfectly equal, as the number of candidates increases, the sum of squared per candidate vote share ( $\sum v_i^2$ ) will decrease, so the *ENC* index will go up. Unfortunately, this increasing competitiveness effect of  $N$  is entirely offset by the  $N$  term that Kenig used to normalize the *ENC* index.
- 4 The between-group variances ( $\sigma_a^2$ ) are assumed to be zero and distributed *I.I.D.*
- 5 This condition is necessary because candidates who are *effective* candidates that enter into our computation cannot receive a vote share smaller than or equal to zero.

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